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Nonlinear analysis of contemporary and historic masonry vaulted elements externally strengthened by FRP

Gehan A. Hamdy*, Osama A. Kamal[b], Mohamed O.R. El-Hariri[b] and Tarik S. El-Salakawy[c]

Department of Civil Engineering, Faculty of Engineering at Shoubra, Benha University, 108 Shoubra St., Shoubra 11241, Cairo, Egypt

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Abstract. This paper addresses numerical modeling and nonlinear analysis of unreinforced masonry walls and vaults externally strengthened using fiber reinforced polymers (FRP). The aim of the research is to provide a simple method for design of strengthening interventions for masonry arched structures while considering the nonlinear behavior. Several brick masonry walls and vaults externally strengthened by FRP which have been previously tested experimentally are modeled using finite elements. Numerical modeling and nonlinear analysis are performed using commercial software. Description of the modeling, material characterization and solution parameters are given. The obtained numerical results demonstrate that externally applied FRP strengthening increased the ultimate capacity of the walls and vaults and improved their failure mode. The numerical results are in good agreement with the experimentally obtained ultimate failure load, maximum displacement and crack pattern; which demonstrates the capability of the proposed modeling scheme to simulate efficiently the actual behavior of FRP-strengthened masonry elements. Application is made on a historic masonry dome and the numerical analysis managed to explain its structural behavior before and after strengthening. The modeling approach may thus be regarded a practical and valid tool for design of strengthening interventions for contemporary or historic unreinforced masonry elements using externally bonded FRP.

Keywords: masonry structure; vault; retrofit/rehabilitation; composites; fiber reinforced; finite element method (FEM); non-linear analysis

1. Introduction

Unreinforced masonry arches and vaults are frequent in many historic structures worldwide. These valuable structures are often subjected to deterioration and misuse and require strengthening. Any intervention strategy should be based on deep understanding of the behavior of the existing structure as well as its behavior after the proposed retrofit measures are made (Croci 1998, Lourenco 2002). Accurate structural analysis of masonry constructions is a true challenge. The mechanical behavior of masonry structural elements exhibits non-homogeneity and directional properties, in addition to cracking due to weakness and brittleness of mortar joints. Linear analysis usually performed for simplifying the analysis and design of masonry structures, where linear isotropic behavior is assumed, often underestimates the structural capacity of such constructions. Nonlinear analysis is reported to describe more accurately the actual behavior and capacity of the structure in many cases (Giordano *et al*. 2002, Lourenco *et al*. 2007).

In this paper finite element modelling and nonlinear analysis are conducted using commercially available computer software ANSYS v.12 (2012). Description of the adopted modelling parameters, material characterization and nonlinear solution parameters is given. Numerical investigation is made for masonry walls and vaults externally strengthened by bonded laminates of Fibre Reinforced Polymers (FRP), which have been experimentally tested by El-Salakawy *et al*. (2014). The obtained numerical results are compared with the experimental results. Application of the adopted procedure is also made on a historic masonry dome to demonstrate the capability of the proposed analysis to explain its structural behavior prior to and after strengthening using FRP composites.

*Corresponding author, Associate Professor
E-mail: gehan.hamdy@feng.bu.edu.eg
[a]Associate Professor.
[b]Professor
[c]Ph.D. Student

2. Numerical modeling and nonlinear analysis

2.1 Approaches for modeling and nonlinear analysis of masonry

Different modeling strategies may be followed to represent the heterogeneous and anisotropic nature of masonry construction using finite elements, depending on the required level of accuracy. These strategies are illustrated in Fig. 1 and may be described as follows (Lourenco 2002, Roca *et al*. 2010).

(a) *Detailed micro-modeling*: both mortar and masonry units are modeled independently as continuum elements where inelastic properties for each are assigned. Discontinuous elements are used to model the interface between mortar and units.

(b) *Simplified micro-modeling*: expanded units are

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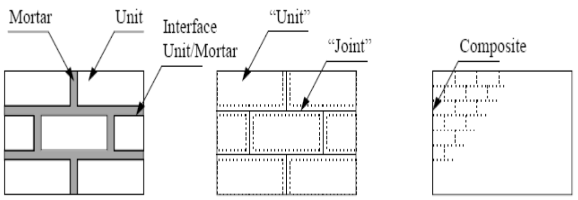
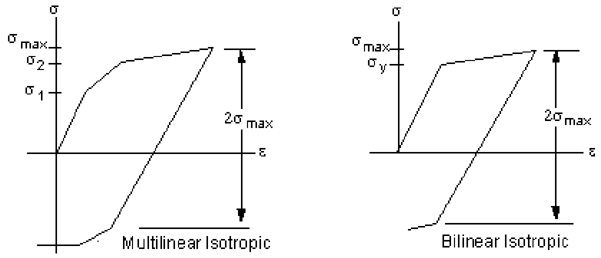


(a) Detailed micro-modeling (b) Simplified micro-modeling (c) Macro-modeling

Fig. 1 Modeling strategies for masonry (Lourenco 2002)



(a) Multilinear for masonry (b) Bilinear for FRP

Fig. 2 The adopted stress-strain curves (ANSYS 2012)

represented by continuum elements where behavior of mortar joints and unit-mortar interface is lumped in discontinuous elements or interface elements. Masonry is considered as a set of elastic blocks bonded by potential fracture/slip lines at the joints.

(c) *Macro-modeling (homogenization theory)*: the masonry units, mortar and mortar-unit interface are smeared out in a homogenous continuum material. Macro models are more applicable when the structure has large dimensions and stresses are uniformly distributed along the macro-length.

Micro modelling needs higher computational effort and is better suited to investigate the local behaviour of load-bearing walls in detail whereas macro modelling is more practice oriented due to reduced time and memory requirements and user-friendly mesh generation and is used to investigate the general behaviour of masonry buildings (Lourenco *et al*. 2007, Roca *et al*. 2010).

Different modeling strategies were applied by Da Porto *et al*. (2010) and Milani (2011) to represent numerically clay masonry walls experimentally tested under in-plane load. It was concluded that detailed micro-models are capable of addressing some of the complexities but the macro modeling approach described the structural behavior with acceptable accuracy.

Finite Element (FE) macro-models were reported to provide accurate simulation of the nonlinear response of masonry structures (Lourenco 2002, Roca *et al*. 2013). Finite element homogenization approach was used for study the seismic behavior of existing and heritage masonry structures (Clementi *et al*. 2016). Mendes and Lourenco (2010) used finite elements to model masonry buildings in Lisbon, Portugal. Nonlinear dynamic analysis with time integration and pushover analysis were performed for seismic assessment. The finite element model was calibrated with the experimental results of a reduced scale shaking table tests. The approach was also used for interpretation of existing crack patterns induced by foundation settlement on old masonry buildings, and was able to predict quite accurately the position of the cracks and give interpretation of the reasons and formation of the crack pattern using fully non-linear finite element code (Acito and Milani 2012).

The material properties to be used in finite element analysis are given based on experimental data. Kaushik *et al*. (2007) conducted a comprehensive experimental study on masonry prisms to formulate compressive stress-strain relationships and propose analytical expressions for the modulus of elasticity of masonry. Xin *et al*. (2017) studied the mechanical properties of masonry under biaxial compressive stress and presented the failure curve in terms of two orthotropic principal stresses. Based on experimental data, the failure criterion of brick masonry has been established in the form of the tensor polynomial.

The Discrete Element Method (DEM) was applied by Reccia *et al*. (2012) to the kinematic limit analysis of out-of-plane loaded masonry walls. Blocks are discretized with triangular rigid FEs and are regarded as rigid bodies connected by zero thickness Mohr-Coulomb-type interfaces. Sarhosis *et al*. (2016) used software based on the Distinct Element Method (DEM), a three-dimensional numerical study has been performed to investigate the in-plane and out-of-plane response of classical columns and colonnades during harmonic and seismic loading excitations. Numerical analysis using combined finite/ discrete element method (FDEM) was performed by Balic *et al*. (2016) to investigate the seismic resistance of dry stone arches under in-plane seismic loading. Incremental dynamic analysis was conducted on twelve types of dry stone arches to observe the collapse mechanisms and determine the seismic resistance. The Applied Element Method (AEM), which combines the FEM with Discrete Element Method, was also used by Karbassi and Lestuzzi (2012) to perform nonlinear dynamic analyses on unreinforced masonry buildings and define spectral-based fragility curves of typical masonry buildings.

To overcome the high computational effort needed by FE models, simplified models were developed based on mechanical bi-dimensional models and equivalent frame models. Nonlinear modeling of masonry buildings is done by assigning lumped plastic hinges to isotropic and homogenous equivalent frame elements. This modeling technique was used by Kheirollahi (2013) for performance assessment of unreinforced brick masonry buildings, where the numerical results and pushover curves provided good agreement with those of a reversed cyclic experiment conducted on a full scale, two-story building. Equivalent frame models were also applied by Chacara *et al*. (2017) to evaluate the seismic response of a brick masonry structure which was tested by shaking table. Doven and Kafkas (2017) investigated the elastic behaviour of load-bearing walls through a micro modelling approach, where brick units were modelled by plane frame elements, horizontal joints were modelled by vertical frame elements with equivalent elasticity modulus and moment of inertia and vertical joints were modelled by horizontal plane truss

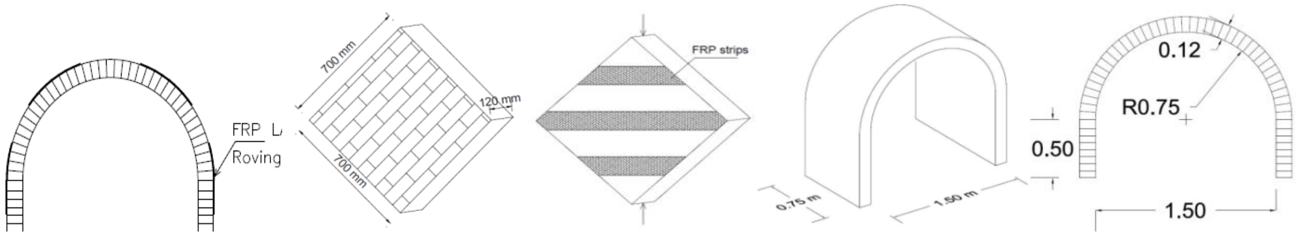
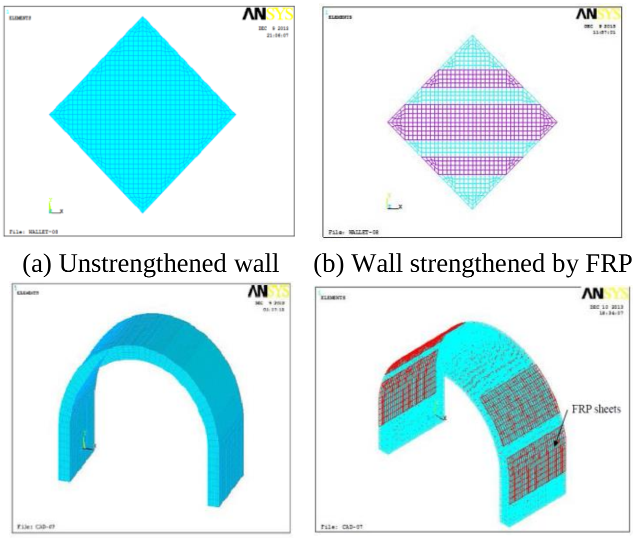


Fig. 3 Dimensions and strengthening for walls and vaults (El-Salakawy *et al.* 2014)



(a) Unstrengthened wall (b) Wall strengthened by FRP

(c) Unstrengthened vault (d) Vault strengthened by FRP

Fig. 4 Finite element 3D model for walls and vaults

elements. The numerically calculated elastic displacements of masonry walls subjected to in-plane static loads were found compatible with published literature.

D'Ambrisi *et al.* (2012) used the nonlinear finite element program ANSYS to carry out nonlinear static and dynamic analyses to study the seismic performances of a medieval tower in Italy. Analytical models of the tower were calibrated on the results of the performed dynamic identification with ambient vibration tests. Kamal *et al.* (2014) used three-dimensional finite elements and nonlinear analysis software ANSYS to represent the nonlinear behaviour of unreinforced masonry structures. An experimental study was conducted in order to validate the accuracy of the adopted modelling and solution procedure. The proposed numerical modelling was concluded to be suitable to study and understand the structural behaviour of existing heritage structures and interpret the cracks or any structural problem encountered.

Kocatürk and Erdoğan (2016) used the nonlinear finite element analysis software ANSYS/LS-DYNA in modelling the minaret of Sultan Ahmed Mosque in Turkey. Nonlinear analysis was conducted for the minaret under some registered earthquake loads to study its stability and demonstrated the importance of the used iron-lead connectors between stones to the energy absorption capacity of the minaret under extensive earthquakes. It was found out that under very big recorded earthquakes, the system plays very important role and helps to keep the structure safe.

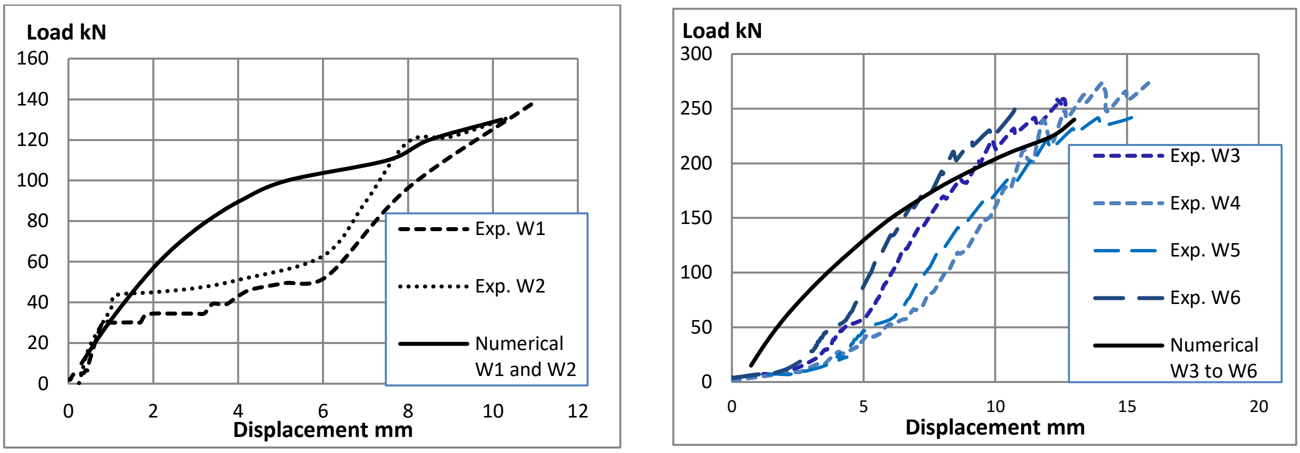
In order to model masonry structures strengthened with FRP, the model should take into account the masonry itself, the FRP reinforcement and the interaction between the masonry and the FRP. Macro-modeling approaches were applied for masonry panels reinforced with FRP strips. Modeling was made by elements having membrane stiffness and tension-only behavior assumed perfectly bonded to the masonry (Gabor *et al.* 2006), or by using special constitutive material models for the masonry-FRP interaction (Grande *et al.* 2013). Homogenized anisotropic material and smeared crack model was used for masonry vaults strengthened with FRP strips at the extrados. FRP strips assumed fully bonded to the masonry were modeled as solid elements with anisotropic material properties (Mahini 2015), or as linear elastic and orthotropic until fiber tensile strength is reached and rupture of fibers occurred (Szolomicki *et al.* 2015). Numerical approaches to model the masonry-FRP interface behavior have been also proposed (Mazotti and Murgo 2015).

2.2 Adopted nonlinear material behavior and solution procedure

Within this research work, macro-modeling is adopted where masonry is considered a homogenous continuum for which the macro behavior is simulated through selection of specific material properties (Milani and Lourenco 2013). The commercial software ANSYS 12.0 (2012) is used for finite element nonlinear analysis. The masonry is modeled as an isotropic material with homogenized properties characterized by different nonlinear softening laws in tension and compression. Multilinear isotropic hardening material is used to simulate the masonry as shown in Fig. 2(a). This type of material (MISO) uses the von Mises yield criteria coupled with an isotropic work hardening assumption described by a multilinear stress-strain curve (Kaushik *et al.* 2007).

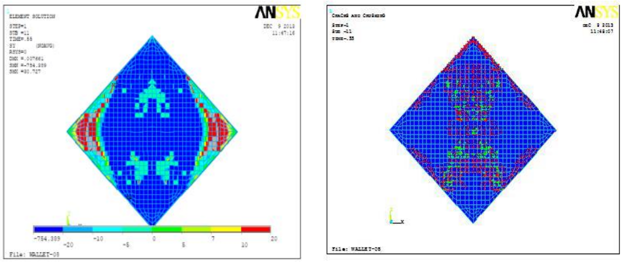
The stress-strain relation for FRP is linear up to failure, thus the bilinear stress-strain relation for steel in Fig. 2(b) is used and yield stress σ_y is given the same value as maximum stress σ_{max}. The user defines material tensile and compressive strengths and shear transfer coefficient (from zero to 1.0). When the solution converges to the cracked state, the modulus and consequently the stiffness normal to the crack face is set to zero.

3. Numerical study



(a) unstrengthened walls (b) FRP-strengthened walls

Fig. 5 Load-displacement curves for walls compared to experimental results



(a) Stresses (S_yx10^{-2} MPa) (b) Crack pattern before failure

Fig. 6 Stresses and crack pattern in the unstrengthened masonry wall

Table 1 Failure loads for walls and vaults

Masonry element	Numerical failure load kN	Experimental failure load kN
Unstrengthened wall	130	135
FRP-strengthened wall	255	259.5
Unstrengthened vault	9	10
FRP-strengthened vault	14	15

For validation of the adopted procedure, numerical modeling and nonlinear analysis were carried out for brick masonry walls and vaults strengthened by externally bonded FRP sheets, which were experimentally tested previously (El-Salakawy *et al*. 2014). The walls and vaults were constructed from commercial perforated clay bricks and cement mortar having compressive strengths 5 MPa and 14.5 MPa. Prism tests were made yielding compressive strength of only 4.45 MPa (El-Salakawy *et al.* 2014).

Masonry walls W1 and W2 were not strengthened, while walls W3 to W6 were strengthened by adhering 200 mm wide strips of GFRP on both sides using polyester resin, and were tested in diagonal compression test until failure. Brick masonry vaults V1, V2 and V3 were not strengthened and vaults V4 and V5 were strengthened using GFRP externally adhered on the extrados at locations chosen near the hinges expected to occur (El-Salakawy *et al*. 2014). The dimensions and strengthening schemes are shown in Fig. 3.

3.1 Numerical model

A three-dimensional finite element mesh is used to simulate the walls and vaults in a macro modeling strategy. Masonry material (bricks and mortar) was discretized using the solid element SOLID65, having eight nodes with three translational degrees of freedom at each node. The model is based upon the crushing and cracking option to locate accurately the cracks (ANSYS 2012). The FRP layer is modeled using the element SHELL63 assumed perfectly bonded to the masonry. The meshes for unstrengthened and strengthened walls and vaults are shown in the Fig. 4.

3.2 Material properties and nonlinear parameters

The material properties of masonry and FRP strengthening are specified based on the experimentally evaluated properties (El-Salakawy *et al*. 2014) and are given as follows.

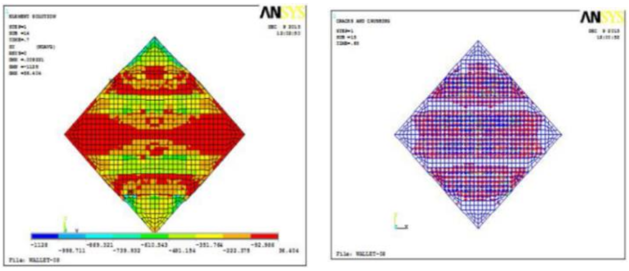
a) Masonry: compressive strength *f'm* = 4.47 MPa (determined experimentally); modulus of elasticity *Em* = 625 MPa, weight density= 16 kN/m^3, major Poisson's ratio = 0.15, tensile strength = 0.447 (value suggested in published research and code recommendations (Kaushik *et al*. 2007, ECP 2005).

b) FRP: ultimate tensile strength = 3800 MPa, modulus of elasticity = 75 GPa (given by manufacturer)

The loading of the model is similar to that conducted in the experimental program (El-Salakawy *et al*. 2014). For nonlinear analysis, iterative solution is adopted with load applied at increments. Nonlinear static analyses are performed and the Newton– Raphson iteration method is adopted by activating the energy norm criterion to check the convergence at each step. The load is applied at increments; made within each load step, the computer program may perform several substeps in which equilibrium iterations are made until convergence criteria are satisfied and a converged solution is reached.

The coefficients and parameters for nonlinear analysis are assigned the following values.

- Shear coefficient along opening cracks (ShrCf-pO) = 0.2



(a) Stresses (S_yx10^{-2}MPa) (b) Crack pattern before failure

Fig. 7 Stresses and crack pattern in the FRP-strengthened masonry wall

- Shear coefficient along closed cracks (ShCf-Cl) = 0.8
- Tension limit, cracking limit (UnTensSt) = 0.425 MPa
- Compression limit, crushing limit (UnCompSt) = 4.25 MPa
- Number of load substeps solution = 10
- Number of equilibrium iterations = 25
- Convergence criteria: Newton-Raphson, displacement control

4. Numerical results and discussion

4.1 Failure loads and load-displacement relations for walls

The failure loads predicted numerically for the walls are given in Table 1, compared to the average experimentally determined values. The load-displacement curves for the unstrengthened walls and for walls strengthened with FRP sheets are shown in Figs. 5(a) and (b), respectively. The numerically predicted load displacement relation gives the same maximum loads and displacements as the experimental values. However, acceptable match is observed for the first and last thirds of the curve, but the middle third shows deviation of the numerically predicted curve from the experimental curves.

Numerical results indicate that strengthening using FRP managed to increase the failure load to about 192% of the control walls ultimate load, and thus is considered an efficient strengthening technique. The final displacement was increased from 11 mm to 16 mm for the strengthened wall, indicating that the strengthening technique improved the deformability of the walls. The results of failure loads and displacement values show that confinement by FRP is an efficient strengthening technique for clay brick masonry walls and assemblages.

4.2 Crack patterns for walls

The numerically evaluated stresses and cracks in unstrengthened and strengthened walls are shown in Figs. 6 and 7, respectively. The crack pattern at failure for the unstrengthened wall shows longitudinal cracks from tip to tip. For the strengthened wall, cracks formed due to crushing at the top part of the wall; also, cracks occurred between the brick elements and the FRP sheets, indicating that the stability of the wall was maintained through the confinement provided by FRP having development length. Failure modes predicted numerically agree with the experimental failure modes observed in Fig. 8(a) and (b) (El-Salakawy *et al*. 2014).

(a) Unstrengthened wall (b) FRP-strengthened wall

Fig. 8 Experimental failure modes for walls (El-Salakawy *et al*. 2014)

4.3 Failure loads and load-displacement relations for vaults

The failure loads evaluated numerically for the vaults are given in Table 1, compared to the values determined experimentally (El-Salakawy *et al.* 2014). Results show that the FRP strengthening technique increased the failure load by about 150% and thus is considered efficient in strengthening masonry vaults. Load-displacement curves for the unstrengthened vaults and for vaults strengthened with FRP are shown in Fig. 9.

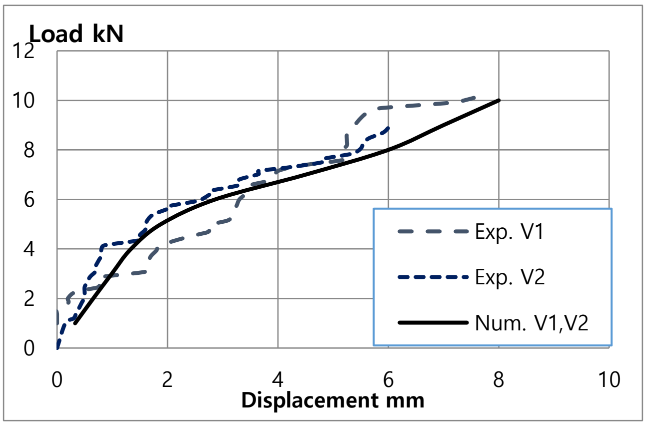
4.4 Crack patterns for vaults

The numerical results for stresses and cracks before failure in the unstrengthened and strengthened vaults are shown in Figs. 10 and 11, respectively. The crack pattern for the unstrengthened vault indicates failure by formation of three hinges. The FRP strengthened vault produced a mode of failure where the crack occurred between the two strengthened zones. Similar crack pattern was observed experimentally as shown in Figs. 10(c) and 11(c).

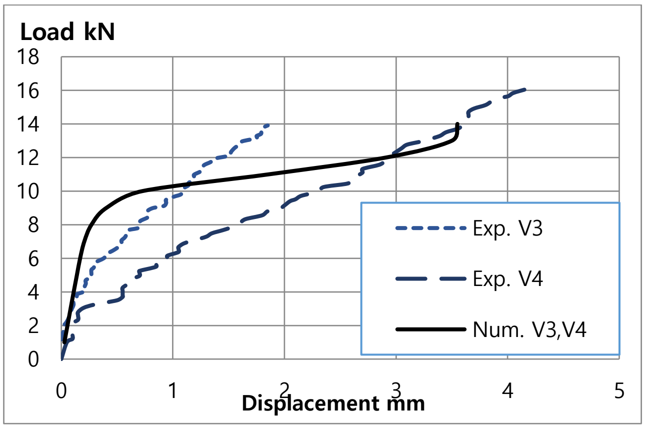
4.5 Comparison between numerical and experimental results

In the current study, the numerically predicted ultimate failure loads and displacements at failure are very close to the experimental results which demonstrate the accuracy of the numerical model. The load-displacement curves show acceptable match for the first and last thirds of the curves but the middle third shows differences between numerical and experimental curves. This deviation may be attributed to non-homogeneity of the experimental samples where defects or weak joints may cause stress concentrations.

Similar results were obtained experimentally by Oliveira *et al*. (2010) where FRP sheets externally applied on the extrados of masonry vaults have increased significantly the load-carrying capacity and modified the collapse

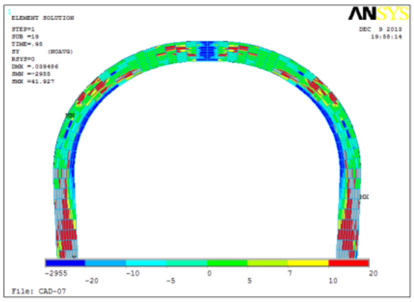


(a) unstrengthened vaults

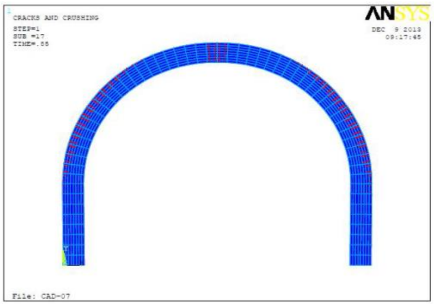


(b) FRP-strengthened vaults

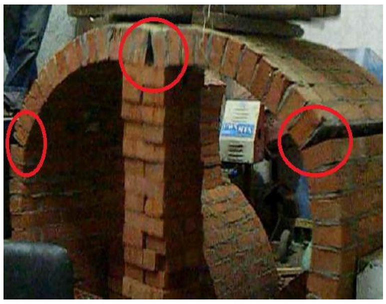
Fig. 9 Load-displacement curves for vaults compared to experimental results



(a) stresses (Syx10^{-2} MPa)

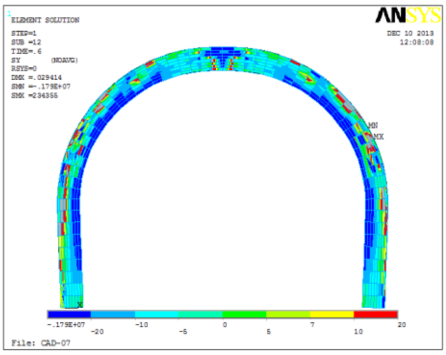


(b) crack pattern before failure

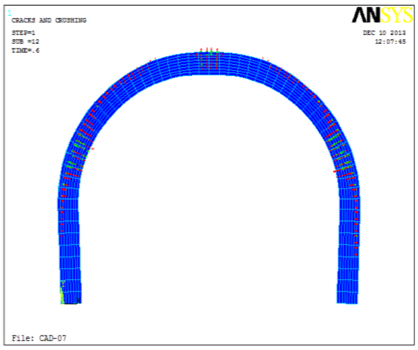


(c) experimental failure mode (El-Salakawy *et al*. 2014)

Fig. 10 Unstrengthened masonry vault: Stresses and crack pattern



(a) Stresses (Syx10^{-2} MPa)



(b) Crack pattern before failure

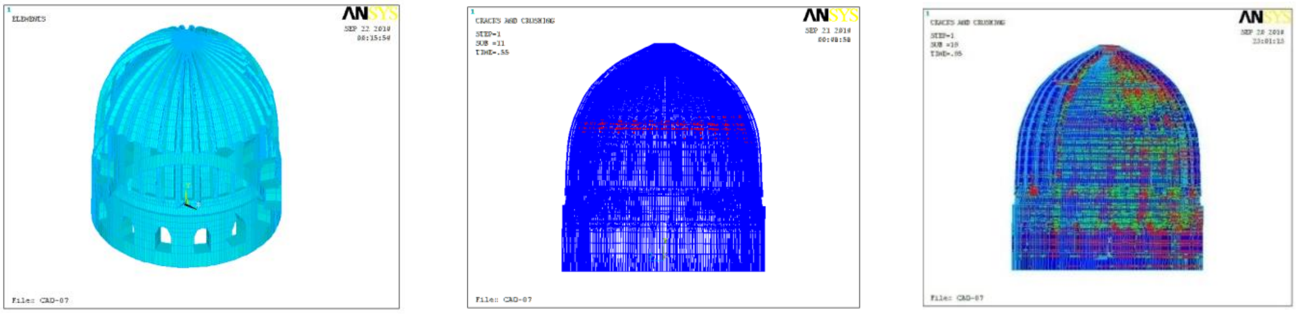
(c) Experimental failure mode (El-Salakawy *et al*. 2014)

Fig. 11 FRP-strengthened masonry vault: Stresses and crack pattern

(a) Exterior view

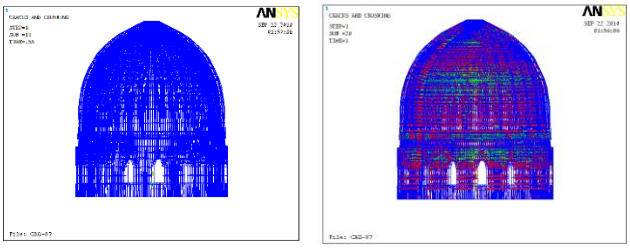
(b) Executed FRP strengthening

Fig. 12 Case study: Historic dome of Sodoun (Mahfouz and Rizk 2003)



(a) Finite element model (b) First cracks at load step 11 (c) Crack pattern before failure

Fig. 13 Numerical modeling and results of the unstrengthened deteriorated dome



(a) Cracks at load step 11 (b) Cracks before failure

Fig. 14 Crack pattern of FRP strengthened deteriorated dome

mechanism. Mahini (2015) demonstrated that application of FRP strips over the inside and outside surfaces of the cross vaults can prevent the cracks opening and the formation of hinges prior to collapse of the structure. Valluzzi *et al*. (2001) concluded that proper location of carbon FRP strips used to strengthen brick masonry vaults can improve the vaults safety.

5. Case study

5.1 Description

The dome of Sodoun, built in 1468 in the historic district in Cairo, showed deterioration and cracks that threatened collapse of the dome. The heritage dome shown in Fig. 12, is made of fired clay bricks and masonry lime mortar, it has diameter 5 m and height 5.5 m. It is built over four limestone walls of thickness 0.6 m and height 8 m. Strengthening was made in the year 2002 using carbon fiber reinforced polymer (CFRP) strips bonded to the interior surface of the dome and to the exterior of the supporting drum (Mahfouz and Rizk 2003).

5.2 Numerical modeling and material properties

Numerical study is performed using the adopted finite element modeling. The masonry dome was modeled by SOLID65 elements, as shown in Fig. 13(a). The strengthening CFRP sheets and strips were modeled using SHELL63 elements. The material properties for the clay brick masonry were estimated through field tests conducted within the restoration project and reported by Mahfouz and Rizk (2003). Masonry compressive strength ($f'm$), modulus of elasticity and major Poisson's ratio were 0.85 MPa, 119 MPa and 0.15, respectively. These values were adopted in the present study for the originally built dome and reduced by 20% to represent material deterioration.

The CFRP strips have tensile strength 2800 MPa and modulus of elasticity=165 GPa, as indicated by manufacturer.

In order to study the performance of the dome, three cases were modeled and analyzed: the original non-cracked dome having the estimated masonry material properties, the deteriorated dome with reduced material properties, and the deteriorated cracked dome strengthened by externally applied CFRP strips. For all the studied cases, the own weight and wind loads were applied in 20 load steps.

5.3 Numerical results and discussion

For the deteriorated dome, results showed first cracking at load step 11, Fig. 13(b), the numerically obtained crack pattern was very similar to the reported observed cracks (Mahfouz and Rizk 2003). By increasing the load, the last load step gave non-converged solution, indicating that material deterioration can lead to total failure. The crack pattern before failure is shown in Fig. 13(c). Analysis of the deteriorated strengthened dome shows that the CFRP strengthening system managed to prevent propagation of the first crack, Fig. 14(a). By increasing the load, the final crack pattern at load step 20 is shown in Fig. 14(b), where a converged solution was reached. In spite of the crack pattern, the dome was still stable and total failure did not occur. This demonstrates the effectiveness of the used strengthening system in avoiding the dome failure and total collapse. The numerical results are described in more details by El Salakawy (2015).

The numerical results demonstrate that the adopted modeling procedure is suitable and sufficiently reliable for studying the structural behavior of existing historical structures and for interpretation of cracking or of any structural problems encountered in them. The finite element model should have input data for material properties (strength and stress-strain behavior) evaluated through onsite structural and/or material investigations including non-destructive or minor-destructive testing onsite and/or in laboratories. The numerical analysis could therefore allow prediction of the behavior of the historical buildings before and after the FRP-strengthening.

6. Conclusions

This paper presents a simplified procedure for numerical modeling and nonlinear analysis of masonry flat and vaulted elements strengthened by externally adhered FRP laminates. The numerical results are compared with published experimental outcomes to investigate the feasibility of the modeling procedure in describing the structural behavior of FRP strengthened brick walls and vaults. Further, application is made on a heritage brick masonry dome which was strengthened by FRP. The main conclusions drawn from this study may be summarized in the following points

• Numerical modeling and nonlinear analysis were done using a commercially available computer program (ANSYS), which renders the approach applicable by practicing designers.

• The numerical results showed good agreement with published experimental results as regards crack patterns, failure mechanisms, maximum load and corresponding maximum deformation.

• Numerical prediction for the load-displacement curve showed acceptable match with the published experimental results except for the middle part of the curve.

• External FRP strengthening nearly doubled the ultimate capacity of the studied unreinforced masonry walls and vaults and improved the failure mode. Other advantages such as ease of installation, small thickness, less intervention and possible removal with less damage (compared to other strengthening schemes such as reinforcement, stitching or anchoring) make FRP an attractive alternative for traditional strengthening methods.

• The adopted numerical modelling may thus be regarded as a reliable tool to explore and compare the effectiveness of FRP strengthening and predict the failure mode, ultimate load-carrying capacity and safety level of existing unreinforced masonry structures.

• Application on a historic dome demonstrated the capability of the adopted procedure to simulate the structural behavior of heritage masonry structures. The model can be efficiently used to justify cracking, predict the failure loads and patterns, and propose and design appropriate strengthening schemes for retrofit, thus preserving and enhancing the safety of distressed heritage structures.

• In this work, full bond is assumed between masonry and FRP laminates. The work may be extended to simulate accurately the bond and transfer of tangential stresses between the masonry and the strengthening elements.

• The research could be extended to study the use of other strengthening techniques such as FRP grids, prestressed FRP laminates, near-surface mounted FRP rods.

References

Acito, M. and Milani, G. (2012), "Homogenization approach for the evaluation of crack patterns induced by foundation settlement on an old masonry building", *Open Civil Eng. J.*, **6**, 215-230.

ANSYS® *Nonlinear Analysis Computer Program*, Release 12.0 (2012), Theory Reference Manual, ANSYS Inc., Canonsburg, PA, U.S.A.

Balic, I., Zivaljic, N., Smoljanovic, H. and Trogrlic, B. (2016), "Seismic resistance of dry stone arches under in-plane seismic loading", *Struct. Eng. Mech.*, **58**(2), 243-257.

Chácara, C., Mendes, N. and Lourenço, P.B. (2016), "Simulation of shake table tests on out-of-plane masonry buildings, part IV: Macro and micro FEM based approaches", *J. Arch. Herit.*, **11**, 103-116.

Clementi, F., Gazzani, V., Poiani, M. and Lenci, S. (2016), "Assessment of seismic behavior of heritage masonry buildings using numerical modeling", *J. Build. Eng.*, **8**, 29-47.

Croci, G. (1998), *Conservation and Structural Restoration of Architectural Heritage*, Advances in Architectures Series, Computational Mechanical Publications, Southampton, U.K.

D'Ambrisi, A., Valentina, M. and Marco, M. (2012), "Seismic assessment of a historical masonry tower with nonlinear static and dynamic analyses tuned on ambient vibration test", *Eng. Struct.*, **36**, 210-219.

Da Porto, F., Guidi, G., Garbin, E. and Modena, C. (2010), "In-plane behavior of clay masonry walls: Experimental testing and finite-element modelling", *J. Struct. Div.*, **136**(11), 1379-1392.

Doven, M.S. and Kafkas, U. (2017), "Micro modelling of masonry walls by plane bar elements for detecting elastic behavior", *Struct. Eng. Mech.*, **62**(5), 643-649.

ECP 204 (2005), *Egyptian Code of Practice for Design and Construction of Masonry Structures*, Ministry of Housing and Urban Communities, Cairo, Egypt.

El-Salakawy, T.S. (2015), "Response of masonry structures strengthened by different techniques", Ph.D. Dissertation, Benha University, Cairo, Egypt.

El-Salakawy, T.S., El-Hariri, O.R., Kamal, O.A. and Hamdy, G.A. (2014), "Experimental investigation of masonry vaults/walls strengthened using different techniques", *J. Civil Eng. Technol.*, **5**(12), 354-365.

Gabor, A., Benanni, A, Jacquelin, E. and Lebon, F. (2006), "Modeling approaches of the in-plane shear behavior of unreinforced and FRP strengthened masonry panels", *Compos. Struct.*, **74**, 277-288.

Giordano, A., Mele, E. and De Luca, A. (2002), "Modeling of historical masonry structures: Comparison of different approaches through a case study", *Eng. Struct.*, **24**(8), 1057-1069.

Grande, E., Imbimbo, M. and Sacco, E. (2013), "Finite element analysis of masonry panels strengthened with FRPs", *Compos. B*, **71**, 1296-1309.

Kamal, O.A., Hamdy, G.A. and El-Salakawy, T.S. (2014), "Nonlinear analysis of historic and contemporary vaulted masonry assemblages", *HBRC J.*, **10**, 235-246.

Karbassi, A. and Lestuzzi, P. (2012), "Fragility analysis of existing unreinforced buildings through a numerical-based methodology", *Open Civil Eng. J.*, **6**, 121-130.

Kaushik, H., Rai, D. and Jain, S. (2007), "Stress-strain characteristics of clay brick masonry under uniaxial compression", *J. Mater. Civil Eng.*, **19**, 728-739.

Kheirollahi, M. (2013), "Equivalent frame model and shell element for modeling of inplane behavior of unreinforced brick masonry buildings", *Struct. Eng. Mech.*, **46**(2), 213-229.

Kocatürk, T. and Erdoğan, Y.S. (2016), "Earthquake behavior of M1 minaret of historical Sultan Ahmed Mosque (Blue Mosque)", *Struct. Eng. Mech.*, **59**(3), 539-558.

Lourenco, P.B. (2002), "Computations on historic masonry structures", *Prog. Struct. Eng. Mater.*, **4**(3), 301-319.

Lourenço, P.B., Milani, G., Tralli, A. and Zucchini, A. (2007), "Analysis of masonry structures: Review of and recent trends of homogenization techniques", *Can. J. Civil Eng.*, **34**, 1443-1457.

Mahfouz, I. and Rizk, T. (2003), "Applications of FRP in

strengthening of structures", *Proceedings of the 6th International Conference on Structural Faults and Repair*, London, U.K., July.
Mahini, S.S. (2015), "Smeared crack material modeling for the nonlinear analysis of CFRP-strengthened historical brick vaults with adobe piers", *Constr. Build. Mater.*, **74**, 201-218.
Mazzotti, C. and Murgo, F.S. (2015), "Numerical and experimental study of GFRP-masonry interface behavior: Bond evolution and role of the mortar layers", *Compos. B*, **75**, 212-225.
Mendes, N. and Lourenço, P.B. (2010), "Seismic assessment of masonry 'Gaioleiro' buildings in Lisbon, Portugal", *J. Earthq. Eng.*, **14**, 80-101.
Milani, G. and Lourenço, P.B. (2013), "Simple homogenized model for the nonlinear analysis of FRP-strengthened masonry structures I: Theory", *J. Eng. Mech.*, **139**(1), 59-76.
Milani, G. (2011), "Simple homogenization model for the non-linear analysis of in-plane loaded masonry walls", *Compos. Struct.*, **89**, 1586-1601.
Oliveira, D.V., Basilio, I. and Lourenço, P.B. (2010), "Experimental behavior of FRP strengthened masonry arches", *J. Compos. Constr.*, **14** (3), 312-322.
Reccia, E., Cazzani, A. and Cecchi, A. (2012), "FEM-DEM modeling for out-of-plane loaded masonry panels: A limit analysis approach", *Open Civil Eng. J.*, **6**, 231-238.
Roca, P., Cervera, M., Gariup, G. and Pelà, L. (2010), "Structural analysis of masonry historical constructions, classical and advanced approaches", *Arch. Comp. Meth. Eng.*, **17**(3), 299-325.
Roca, P., Cervera, M., Pelà, L., Clemente, R. and Chiumenti, M. (2013), "Continuum FE models for the analysis of mallorca cathedral", *Eng. Struct.*, **46**, 653-670.
Sarhosis, V, Asteris, P.G., Mohebkhah, A., Xiao, J. and Wang, T. (2016), "Three-dimensional modelling of ancient colonnade structural systems subjected to harmonic and seismic loading", *Struct. Eng. Mech.*, **60**(5), 633-653.
Szołomicki, J., Berkowski, P. and Barański, J. (2015), "Computer modeling of masonry cross vaults strengthened with fiber reinforced polymer strips", *Arch. Civil Mech. Eng.*, **15**(3), 751-766.
Valluzzi, M.R., Valdemarca, M. and Modena, C. (2001), "Behaviour of brick masonry vaults strengthened by FRP laminates", *J. Compos. Constr.*, **5**(3), 165-169.
Xin, R., Yao, J. and Zhao, Y. (2017), "Experimental research on masonry mechanics and failure under biaxial compression", *Struct. Eng. Mech.*, **61**(1), 161-169.

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